

Enrollment No: _____

Exam Seat No: _____

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Numerical Analysis

Subject Code: 4SC03MTE1

Branch: B.Sc. (Mathematics, Physics)

Semester: 3 Date: 08/12/2015 Time: 02:30 To 05:30 Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
 - (2) Instructions written on main answer book are strictly to be obeyed.
 - (3) Draw neat diagrams and figures (if necessary) at right places.
 - (4) Assume suitable data if needed.
-

Q-1

Attempt the following questions:

(14)

- a) Subtract the approximate numbers 57.4662 from 786.85.
- b) Round off the following numbers correct to four significant figures:
i) 5.2056, ii) 0.0055672
- c) Construct a difference table for $y = x^3 + 2x + 1$, for $x = 1, 2, 3, 4, 5$.
- d) Prove that $\Delta \cdot \nabla = \Delta - \nabla$.
- e) Write Lagrange's inverse interpolation formula.
- f) Construct a divided difference table for the following data

x	5	15	22
y	7	36	160

- g) If $y = 4x^6 - 5x$, find the percentage error in y at $x = 1$, if the error in $x = 0.04$.
- h) Prove that $\mu = \frac{1}{2} \left[E^{\frac{1}{2}} + E^{-\frac{1}{2}} \right]$.
- i) Write Stirling's interpolation formula.
- j) Bessel's formula is most appropriate when p lies between.....
- k) Gauss forward interpolation formula involves odd difference below the central line and even differences on the central line. Determine whether the statement is True or False?



- l) The n^{th} divided difference of a polynomial of degree n is zero. Determine whether the statement is True or False?
- m) Divided differences are not symmetric functions of their argument. Determine whether the statement is True or False?
- n) If the interval of differencing be unity, then $\Delta^n [x]^n = n!$. Determine whether the statement is True or False?

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a) If $u = \frac{5x^3z^3}{3y^2}$ and error x, y, z are 0.001, 0.0001, 0.000001. Find relative maximum error in u at $x = 1, y = 2$ and $z = 3$. (05)

- b) Determine $y(12)$ by Lagrange interpolation from the following values. (05)

x	11	13	14	18	20	23
y	25	47	68	82	102	124

- c) Write down the approximate representation of $\frac{2}{3}$ correct to four significant figures and then find: i) Absolute error, ii) Relative error, iii) Percentage error. (04)

Q-3 Attempt all questions (14)

- a) Given the following table: (05)

x	0	5	10	15	20
$f(x)$	1.0	1.6	3.8	8.2	15.4

Construct the difference table and compute $f(21)$ by using Newton's Backward formula.

- b) Using Newton's divided difference formula, find $f(x)$ from the following data (05)

x	0	2	3	4	6	7
$f(x)$	0	8	0	-72	0	1008

- c) Show that (04)

i) $\Delta[f(x) \cdot \phi(x)] = f(x+h) \cdot \Delta\phi(x) + \phi(x) \cdot \Delta f(x),$
 ii) $\Delta \left[\frac{f(x)}{\phi(x)} \right] = \frac{\phi(x) \cdot \Delta f(x) - f(x) \cdot \Delta \phi(x)}{\phi(x+h)\phi(x)}, [\phi(x) \neq 0]$



Q-4 **Attempt all questions** (14)

a) Show that : (05)

i) $\Delta^n u_{x-n} = u_x - n u_{x-1} + \frac{n(n-1)}{2} u_{x-2} - \dots + (-1)^n u_{x-n},$

ii) $e^x \left(u_0 + x\Delta u_0 + \frac{x^2}{2!} \Delta^2 u_0 + \dots \right) = u_0 + x u_1 + \frac{x^2}{2!} u_2 + \dots$

b) From the following table, estimate the number of students who obtain mark 40 to 45 (05)

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	42	51	35	31

c) Find $f(x)$, when its first difference is $x^3 + 4x^2 + 2x + 7$. (04)

Q-5 **Attempt all questions** (14)

a) State and prove Newton's forward interpolation formula. (07)

b) From the following table, find the value of $e^{1.17}$ using Gauss's backward interpolation formula. (05)

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

c) Show that $\Delta^n [k e^{ax}] = k (e^{ah} - 1)^n e^{ax}$. (02)

Q-6 **Attempt all questions** (14)

a) State and prove Bessel's interpolation formula. (07)

b) From the following table, find the value of x when $y = 13.5$, by using inverse interpolation formula (05)

x	93	96.2	100.0	104.2	108.7
y	11.38	12.80	14.70	17.07	19.91

c) Prove that $\Delta \log f(x) = \log \left[1 + \frac{\Delta f(x)}{f(x)} \right]$. (02)

Q-7 **Attempt all questions** (14)

a) State and prove Newton's Divided difference interpolation formula. (05)

b) Use Lagrange's formula to express the function $\frac{x^2+x-3}{x^3-2x^2-x+2}$ as a sum of partial fractions. (05)

c) Evaluate the missing terms in the following table. (04)



x	0	1	2	3	4	5
$f(x)$	0	?	8	15	?	35

Q-8

Attempt all questions

(14)

a) State and prove Lagrange's interpolation formula.

(05)

b) From the following table, find $f(34)$ using Laplace Everett's formula.

(05)

x	20	25	30	35	40
$y = f(x)$	11.4699	12.7834	13.7648	14.4982	15.0463

c) If $f(x) = \frac{1}{x^2}$, find the divided differences $[a, b]$ and $[a, b, c]$.

(04)

